

STABILIZATION OF THE BUNCH LENGTHENING IN STORAGE RING

A.V. BUROV, A.A. ZHOLENTS

Institute of Nuclear Physics, 630090, Novosibirsk, USSR

Abstract. The method of amplifying the longitudinal focusing is considered as a means against the lengthening effect of an intense bunch in the storage ring. Main limitations of this approach are analyzed. A conclusion for having a big momentum compaction factor to obtain a small longitudinal emittance is made.

PROPOSAL

To provide a high luminosity on the colliding beam facility it is important to have the bunch length smaller than the value of β -function at the collision point, which in its turn is also advantage to be made as small as possible. The obtaining of a short length for intense bunch is hindered by the bunch lengthening effect, when the length is determined from a well known equation¹:

$$\sigma^3 = \frac{Ne^2 Rc}{\sqrt{2\pi} \kappa} \left| \frac{Z}{n} \right|. \quad (1)$$

Here and below CGS system and the following designations are used: σ is r.m.s. longitudinal particle deviation from the bunch mass centre; N is a number of particle in a bunch; R —average machine radius, $R = \Pi/2\pi$, where Π is the circumference; Z —broad-band impedance of the vacuum chamber; n —azimuthal harmonic number, $n = R/\sigma$; κ —rigidity of RF focusing, determined so, that a particle with a deviation x from the equilibrium receives an energy κx per one revolution. For N_R cavities with a length $\lambda/2$ and an amplitude of the electric field on the axis \mathcal{E}_R

$$\kappa = 2e\mathcal{E}_R N_R. \quad (2)$$

Synchrotron oscillation frequency

$$\Omega_s^2 = \frac{\kappa \alpha c}{ET}.$$

E is the equilibrium particle energy in a storage ring; T is the revolution period; α is the momentum compaction factor.

Substituting (2) into (1) one can get the number of cavities required to provide the bunch length of σ .

$$N_R = \frac{NeRc \left| \frac{Z}{n} \right|}{2\sqrt{2\pi} \mathcal{E}_R \sigma^3}. \quad (3)$$

The data obtained from measurements of vacuum chamber impedance for a number of

storage rings indicate that for the case of a carefully smoothed vacuum chamber one can expect a value of the given impedance²

$$\left| \frac{Z}{n} \right| = (1-3) \Omega.$$

Assuming in (3) $\left| \frac{Z}{n} \right| = 1.5\Omega$, $N = 6 \cdot 10^{11}$, $\mathcal{E}_R = 6 \text{ MV/m}$, $\Pi = 600 \text{ m}$, to provide $\sigma = 1 \text{ cm}$ one should have

$$N_R = 140.$$

In Reference³ a class of broadband impedance is found which with the growth of N does not correspond to the swelling of the bunch, but, on the contrary, to its shortening. However, the uncertainty in the real utilization of such a shortening broadband system makes one to search for new means to obtain a short bunch length.

The only means left as it seems is an increase on the focusing rigidity κ . After attaining the maximum of \mathcal{E}_R it should be done by increasing on the number of cavities. That does not mean that all cavities should be obligatory active. A high-quality passive cavity with a right tuned eigenfrequency is also known to focus a bunch. Since both the effects—the bunch lengthening (1) and the focusing by passive cavities—are proportional to the number of particles in the bunch, one can suspect that the bunch length selfstabilization will take place at the intensities over the lengthening threshold. The present paper gives a detailed consideration of this proposal and related problems.

LONGITUDINAL DYNAMICS

Synchrotron oscillations of electrons in the storage ring with M bunches, each containing N particles, are determined by the accelerating voltage of active cavities and the total field, induced by all bunches in passive cavities. The equations of synchrotron oscillations have the form:

$$\begin{aligned} \dot{x} &= -\alpha c \varepsilon, \\ \dot{\varepsilon} &= \frac{\omega_s^2}{\alpha c} x - \frac{Nc^2}{ET} \sum_{k=-\infty}^{\infty} \tilde{W} \left[kT_1 - \frac{x}{c} \right]. \end{aligned} \quad (4)$$

Here x is the longitudinal displacement of electron, ε is the relative energy shift, $\tilde{W}(t)$ is the wake function of the cavity series, T_1 is the bunch pass period and ω_s is the synchrotron oscillations frequency in the absense of passive cavities.

From Eq. (4) the total frequency of synchrotron oscillations is determined:

$$\Omega_s^2 = \omega_s^2 - i \frac{Ne^2 \alpha M}{ET^2} \sum_{p=-\infty}^{\infty} (p\omega_1) \tilde{Z}(p\omega_1). \quad (5)$$

Here

$$\tilde{Z}(\omega) = \int_0^{\infty} \tilde{W}(t) e^{i\omega t} dt \quad (6)$$

is the narrow-band impedance, $\omega_1 = M\omega_0$ is the bunch pass frequency.

In the case of a narrow-band impedance with a small tune shift of the eigenfrequency with respect to integer multiple of ω_1 one only consider in sum (5) the terms in the vicinity of pole Z : $p = \pm p_0$, $p_0\omega_1 = \Omega$; $\Omega \simeq \omega_R$.

The wake function of a cylindrical single-mode cavity with a radius b and a length g has the form:

$$W(t) = W_0 e^{-\gamma_R t} \cos \omega_R t, \quad (\tilde{W} = N_R W), \quad (7)$$

$$\omega_R = \frac{2.4c}{b}, \quad (8)$$

$$W_0 = \frac{12g|\theta|^2}{b^2}, \quad (9)$$

$$\theta = \frac{\exp\left[i\omega_R \frac{g}{c}\right] - 1}{\omega_R \frac{g}{c}}. \quad (10)$$

The corresponding narrow-band impedance is equal to:

$$Z(\Omega) = W_0 \frac{i\Omega}{\Omega^2 - \omega_R^2 + 2i\gamma_R \Omega}. \quad (11)$$

Substituting (11) into (5), one can obtain:

$$\Omega_s^2 = \omega_s^2 + \frac{\alpha N e^2 W_0 N_R M}{ET^2} \frac{\omega_R}{\Delta\Omega}, \quad (12)$$

where $\Delta\Omega = \Omega - \omega_R$.

From here it follows, that the condition:

$$\Delta\Omega > 0 \quad (13)$$

satisfies the focusing action of passive cavities. The amplitude of the longitudinal electric field on the cavity axis is

$$\mathcal{E}_R = \frac{Ne W_0 M}{Tg|\theta|\Delta\Omega} = 0.8 \frac{MNe|\theta|}{Rb} \frac{\omega_R}{\Delta\Omega} \quad (14)$$

and it is convenient to express the synchrotron oscillation frequency in terms of this amplitude

$$\Omega_s^2 = \omega_s^2 + \frac{\alpha e \mathcal{E}_R g |\theta| N_R \omega_R}{ET}. \quad (15)$$

Let us now consider the multibunch coherent oscillations. The motion of short bunches, as compared to the cavity length, is described by the system of equations:

$$\begin{aligned} \dot{x}_\mu(qT) &= -\alpha c \epsilon_\mu(qT), \\ \dot{\epsilon}_\mu(qT) &= \frac{\omega_s^2}{\alpha c} x_\mu(qT) - \frac{Ne^2}{TE} \sum_{v,k} \tilde{W} \left[(q-k)T + \frac{\mu-v}{M} T - \frac{x_\mu(qT) - x_v(qT)}{c} \right]. \end{aligned} \quad (16)$$

Here x_μ is the longitudinal shift of the electron in the μ -bunch ($\mu=0, 1, \dots, M-1$), ϵ_μ is its relative energy shift, q is the revolution number. From here the eigenfrequencies of the multibunch motion are determined

$$\omega^2 = \omega_s^2 - \frac{iMNe^2\alpha}{ET^2} \sum_p [p\omega_1 \tilde{Z}(p\omega_1) - (p\omega_1 - l\omega_0 + \omega) \tilde{Z}(p\omega_1 - l\omega_0 + \omega)]. \quad (17)$$

Here l is the number of the multibunch mode ($l=0, 1, \dots, M-1$). The mode $l=0$ is stable when condition (13) is satisfied. The modes $l \neq 0$ in correspondence with (17) are unstable. The instability increment quickly decreases with the number of l .

$$\operatorname{Im}(\omega) = \frac{MNe^2 \alpha \omega_R N_R \gamma_R \Delta\Omega}{2\pi b E \omega_0 l^3}. \quad (18)$$

For a quantitative estimation of the increment let us calculate the tune shift $\Delta\Omega/\omega_R$ and the synchrotron frequency using formulae (14), (15). Assuming the electric field amplitude for a superconductive cavity to be 6 MV/m and $N=6 \cdot 10^{11}$, $M=20$, $g=30$ cm, $b=23$ cm ($\lambda=60$ cm), $\Pi=600$ m, $E=5$ GeV, $N_R=140$, $\alpha=5 \cdot 10^{-3}$, one can find

$$\frac{\Delta\Omega}{\omega_R} = 7 \cdot 10^{-5}, \quad \frac{\Omega_s}{\omega_0} = 0.16.$$

Assuming $\gamma_R/\omega_R=5 \cdot 10^{-10}$, one can obtain

$$\operatorname{Im}[\omega(l=1)] = 0.11 \text{ s}^{-1}.$$

Such a small value of the increment indicates the real absence of corresponding coherent effects.

According to (16) the power of coherent losses in the narrowband system is

$$P_R = \frac{M^2 N^2 e^2 W_0 N_R}{T^2} \frac{\gamma_R}{\Delta\Omega^2} = \frac{M^2 N^2 c^2 N_R}{2.4 RT} \frac{\gamma_R \omega_R}{\Delta\Omega^2}. \quad (19)$$

For the values of the storage ring parameters mentioned above, it is

$$P_R = 1.1 \text{ kW}$$

LIMITATIONS OF THE CAVITIES NUMBER

By increasing the number of cavities in the storage ring we simultaneously increase the broadband longitudinal impedance. If the impedance introduced does not exceed the parasitic impedance of the machine, the bunch length can be stabilized. Otherwise the lengthening will occur already at the introduced impedance. This limits the maximum number of cavities:

$$N_R < N_1 = \frac{|Z|}{|Z_\omega|}, \quad (20)$$

Z_ω is the broadband longitudinal impedance of the cavity at the harmonic $n=R/\sigma^4$:

$$|Z_\omega| = 50 \frac{\sqrt{g\sigma}}{a} \Omega.$$

Here a is the radius of the tube connecting the cavities. At $|Z|/n=1.5\Omega$, $g=30$ cm, $a=7.5$ cm and $\sigma=1$ cm, one can obtain

$$N_1 = 400.$$

The second limitation for the number of cavities is connected to the power of coherent losses at their broadband impedance. Since the energy loss of a single cavity is equal to⁴:

$$P_1 = \frac{2M(Ne)^2}{\pi Ta} \sqrt{\frac{g}{\sigma}} \quad (21)$$

then, with the RF power P available for compensating these losses, we come to the fol-

lowing limitations:

$$N_R < N_2 = \frac{P}{P_1}. \quad (22)$$

Assigning $P = 2.5$ MW, one can find

$$N_2 = 65.$$

The RF power, required for providing the length σ of a bunch with N particles, is determined by (3) and (21).

$$P_z = \frac{2M(Ne)^3 c^2 \left| \frac{Z}{n} \right| \sqrt{g}}{(2\pi)^{5/2} \sigma^{7/2} \mathcal{E}_R a}. \quad (23)$$

The third limitation for the number of cavities follows from the analysis of stability of transverse coherent oscillations. The distortion of the longitudinal potential well results in a large frequency spread

$$\Delta\Omega_s \simeq \Omega_s.$$

Due to this fact the slow coherent oscillations compared to the synchrotron frequency are stable. An approximated condition for the absence of fast instabilities can be obtained, for example, in the model with two macroparticles⁵:

$$\frac{Ne^2 R \omega_0^2}{8\omega_\beta \Omega_s E} \bar{G}_\perp N_R \leq 2, \quad (24)$$

where \bar{G}_\perp is the average value of the single cavity transverse wake function $G_\perp(s)$, ω_β is the betatron oscillations frequency. According to the results of Reference⁴:

$$G_\perp(s) = \frac{5}{\pi} \frac{\sqrt{2gs}}{a^3}.$$

Assuming $\bar{G}_\perp \simeq G_\perp(2\sigma)$, $\Omega_s/\omega_0 = 0.16$, $\omega_\beta/\omega_0 = 20$ and for the same values of N , R , E and a as presented above, it follows from (24) that to provide the transverse stability, the following condition should be satisfied

$$N_R < N_3 = 10^4. \quad (25)$$

Of all the limitations for the number of cavities considered above, condition (22) seems most important.

LONGITUDINAL EMITTANCE

It often occurs that not only a short bunch, but also a small longitudinal emittance in the whole is required. For instance, this problem is significant for damping rings, producing beams for linear colliders and storage ring factories worked on the narrow resonances. The longitudinal emittance $\sigma\sigma_\epsilon$, of a bunch with the number of particles over the lengthening threshold is calculated from (1) – (3):

$$\sigma\sigma_\epsilon = \frac{1}{2\pi} \left(\frac{\Pi}{2e \mathcal{E}_R N_R} \right)^{1/6} \left(Ne^2 \left| \frac{Z}{n} \right| c \right)^{2/3} \frac{1}{\sqrt{\alpha E}}. \quad (26)$$

It follows from here that the choice of the storage ring optics with a big momentum

compaction factor promotes the achievement of the ultimately small longitudinal emittance. The above formula gives the value of emittance, when the bunch lengthening is determined entirely by a microwave instability. If the lengthening (1) is also caused by the distortion of the potential well, which is not accompanied by an increase in the energy spread, then the longitudinal emittance will be lower.

REFERENCES

1. L.J. Laslett, V.K. Neil and A.M. Sessler. Rev. of Sci. Inst., N3, 32 (1961); D. Boussard. CERN Lab./RF/INT/75-2, 1975.
2. L. Rivkin et al. Bunch Lengthening in the SLC Damping Rings. SLAC-PUB-4656. EPAC Conf. Rome, 1988;
K.L.F. Bane. The calculated Longitudinal Impedance of the SLC Damping Rings. SLAC-PUB-4618, EPAC Conf. Rome, 1988.
3. A.V. Burov. Bunch Lengthening— is it Inevitable? INP Preprint 89-12 (Novosibirsk, 1989); Proc. of this Conf.
4. A.V. Novokhatski. On the Estimation of the Wake Potential for an Ultrarelativistic Charge in an Accelerating Structure. INP Preprint 88-39 (Novosibirsk, 1988).
5. A. Chao. In Physics of High Energy Particle Accelerators. AIP Conf. Proc., N 105 (New York, 1983).